Proceeding of the International Seminar and Conference 2015: The Golden Triangle (Indonesia-India-Tiongkok) Interrelations in Religion, Science, Culture, and Economic.
University of Wahid Hasyim, Semarang, Indonesia. August 28-30, 2015
Paper No. B.4

Linear Consecutive $k-out-of-n$ Unrepairable System

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Abstract-The consecutive $k-out-of-n$ repairable system has always been very populer models in reliability mathematics. It has been used to model various engineering systems, such as street-lamp system, microwave stations of a telecom network, oil pipeline system, and other engineering fields. This paper researched a linear consecutive $k-out-of-n$ unrepairable system. Assuming the system has no repairman, and the working time of each component is negative exponentially distributed and independent of each other. Firstly, study the liner consecutive four out of eight fail system in the particular example. Then we discuss two kinds of three and two units out of eight units fail, the system cannot work. Finally, according to the relationship of the n and k, the system will be discussed three cases to obtain failed probability and reliability by using total probability formula.

Key words: $k-out-of-n$ unrepairable system, key component.

1. Introduction

An n component system that works (or is “good”) if and only if at least k of the n components work (or are good) is called a $k-out-of-n:G$ system. An n component system that fails if and only if at least k of the n components fail is called a $k-out-of-n:F$ system. Based on these two definitions, a $k-out-of-n:G$ system is equivalent to an $(n-k+1)-out-of-n:F$ system. The term k-out-of-n system is often used to indicate either a G system or an F system or both. Since the value of n is usually larger than the value of k, redundancy is generally built into a $k-out-of-n$ system. Both parallel and series systems are special cases of the $k-out-of-n$ system. A series system is equivalent to a $1-out-of-n:F$ system and to an $n-out-of-n:G$ system while a parallel system is equivalent to an $n-out-of-n:F$ system and to a $1-out-of-n:G$ system.

The $k-out-of-n$ system structure is a very popular type of redundancy in fault tolerant systems. It is well known that a microwave signal transmitting system is composed of many relay stations, and these relay stations are arranged linearly (circularly). Usually, people use the redundancy design to improve the reliability of the system, a parallel system is just a typical redundancy system. Microwave signal transmitting systems are often designed using the redundancy method, that is, each relay station can transmit a signal reaching the next k relay stations. Thus, the system fails if and only if at least k adjacent relay stations fail in the system, it is named consecutive $k-out-of-n:F$ system. Now, we consider a more complex system, that is named as m consecutive $k-out-of-n:F$ system, the consecutive $k-out-of-n:F$ system is its special case (m = 1). If we consider that components are repairable, then such system is called a repairable m consecutive $k-out-of-n:F$ system. In this paper, we consider only a linear system.

Among applications of the $k-out-of-n$ system model, the design of electronic circuits such as very large scale integrated (VLSI) and the automatic repairs of faults in an online system would be the most conspicuous. This type of system demonstrates what is called the voting redundancy. In such a system, several parallel outputs are channeled through a decision-making device that provides the required system function as long as at least a predetermined number k of n parallel outputs are in agreement.

In this paper, we provide a detailed coverage on reliability evaluation of the linear consecutive $k-out-of-n$ unrepairable systems. Firstly, study the liner consecutive four out of eight fail system in the particular example. Then we discuss two kinds of three and two units out of eight units fail, the system cannot work. Finally, according to the relationship of the n and k , the system will be discussed three cases to obtain failed probability and reliability by using total probability formula.

2. Assumptions

The system is composed of n component parts of the same model, the i part is expressed by B_i $(i=1,2,...,n)$ and in order to facilitate the discussion, the following hypothesis is made:

(1) Components are beyond repair after failure;
(2) The failure circumstances in the totals $n-k+1$;
(3) A_i indicates that there are successively $B_iB_{i+1}...B_{i+k+1}$ failure circumstances; $(i=1,2,...,n-k+1)$
(4) n components of the same model comply with the distribution of negative exponential λ , and every component is independent from each other.

3. Reliability Analysis of Special Circumstances

According to the above assumptions, the probability of A_1 :

$$P(A_1)=\left[\int_0^t\left(\lambda e^{-\lambda t}\right)dt\right]^k=\left(1-e^{-\lambda t}\right)^k,(i=1,2,\cdots,n-k+1)$$

The Formula of System failure probability:

$$P(A_1\cup A_2\cup A_3\cup...\cup A_{n-k+1})$$
$$=\sum_{i=1}^{n-k+1}P(A_i)-\sum_{i=1}^{n-k}\sum_{j=2,j>i}^{n-k+1}P(A_iA_j)+\sum_{i=1}^{n-k-1}\sum_{j=2,j>i}^{n-k}\sum_{l=3,l>j}^{n-k+1}P(A_iA_jA_j)$$
$$-...+(-1)^{n-k}P(A_1A_2A_3...A_{n-k}A_{n-k+1})$$

Now, $n=8$, $k=4$; $n=8$, $k=3$; $n=8$, $k=2$ three circumstances are discussed.

(1) $n=8$, $k=4$

According to the definition and the above assumptions, the system has five kinds of failure circumstances:

$$A_1(B_1B_2B_3B_4),A_2(B_2B_3B_4B_5),A_3(B_3B_4B_5B_6),A_4(B_4B_5B_6B_7),A_5(B_5B_6B_7B_8)$$

$$P(A_1)=P(A_2)=P(A_3)=P(A_4)=P(A_5)=\left(1-e^{-\lambda t}\right)^4$$

$$P(A_1A_2)=P(A_2A_3)=P(A_3A_4)=P(A_4A_5)=\left(1-e^{-\lambda t}\right)^5$$

$$P(A_1A_3)=P(A_2A_4)=P(A_3A_5)=\left(1-e^{-\lambda t}\right)^6$$

$$P(A_1A_4)=P(A_2A_5)=\left(1-e^{-\lambda t}\right)^7$$

$$P(A_1A_5)=\left(1-e^{-\lambda t}\right)^8$$

$$P(A_1A_2A_3)=P(A_2A_3A_4)=P(A_3A_4A_5)=\left(1-e^{-\lambda t}\right)^6$$

$$P(A_1A_2A_4) = P(A_1A_3A_4) = P(A_2A_3A_5) = P(A_2A_4A_5) = \left(1-e^{-\lambda t}\right)^7$$

$$P(A_1A_2A_5) = P(A_1A_3A_5) = P(A_1A_4A_5) = \left(1-e^{-\lambda t}\right)^8$$

$$P(A_1A_2A_3A_4) = P(A_2A_3A_4A_5) = \left(1-e^{-\lambda t}\right)^7$$

$$P(A_1A_2A_3A_5) = P(A_1A_2A_4A_5) = P(A_1A_3A_4A_5) = \left(1-e^{-\lambda t}\right)^8$$

$$P(A_1A_2A_3A_4A_5) = \left(1-e^{-\lambda t}\right)^8$$

The system failure probability P(t) and reliability system R(t) are

$$P(A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5)$$

$$= 5\left(1-e^{-\lambda t}\right)^4 - 4\left(1-e^{-\lambda t}\right)^5 - 3\left(1-e^{-\lambda t}\right)^6 - 2\left(1-e^{-\lambda t}\right)^7 - 1\left(1-e^{-\lambda t}\right)^8 +$$

$$3\left(1-e^{-\lambda t}\right)^6 + 4\left(1-e^{-\lambda t}\right)^7 + 3\left(1-e^{-\lambda t}\right)^8 - 2\left(1-e^{-\lambda t}\right)^7 - 3$$

$$\left(1-e^{-\lambda t}\right)^8 + \left(1-e^{-\lambda t}\right)^8$$

$$R(t) = 1 - 5\left(1-e^{-\lambda t}\right)^4 + 4\left(1-e^{-\lambda t}\right)^5 + 3\left(1-e^{-\lambda t}\right)^6 + 2\left(1-e^{-\lambda t}\right)^7 + 1\left(1-e^{-\lambda t}\right)^8 -$$

$$3\left(1-e^{-\lambda t}\right)^6 - 4\left(1-e^{-\lambda t}\right)^7 - 3\left(1-e^{-\lambda t}\right)^8 + 2\left(1-e^{-\lambda t}\right)^7 + 3$$

$$\left(1-e^{-\lambda t}\right)^8 - \left(1-e^{-\lambda t}\right)^8$$

(2) $n=8, \quad k=3$

According to the definition and the above assumptions, the system has six kinds of failure circumstances:

$$A_1(B_1B_2B_3), A_2(B_2B_3B_4), A_3(B_3B_4B_5), A_4(B_4B_5B_6), A_5(B_5B_6B_7), A_6(B_6B_7B_8)$$

$$P(A_1) = P(A_2) = P(A_3) = P(A_4) = P(A_5) = P(A_6) = \left(1-e^{-\lambda t}\right)^3$$

$$P(A_1A_2) = P(A_2A_3) = P(A_3A_4) = P(A_4A_5) = P(A_5A_6) = \left(1-e^{-\lambda t}\right)^4$$

$$P(A_1A_3) = P(A_2A_4) = P(A_3A_5) = P(A_4A_6) = \left(1-e^{-\lambda t}\right)^5$$

$$P(A_1A_4) = P(A_1A_5) = P(A_1A_6) = P(A_2A_5) = P(A_2A_6) = P(A_3A_6) = \left(1-e^{-\lambda t}\right)^6$$

$$P(A_1A_2A_3) = P(A_2A_3A_4) = P(A_3A_4A_5) = P(A_4A_5A_6) = \left(1-e^{-\lambda t}\right)^5$$

$$P(A_1A_2A_4) = P(A_1A_3A_4) = P(A_2A_3A_5) = P(A_2A_4A_5) = P(A_3A_4A_6) = P(A_3A_5A_6) = \left(1-e^{-\lambda t}\right)^6$$

$$P(A_1A_2A_5) = P(A_1A_2A_6) = P(A_1A_3A_5) = P(A_1A_4A_5) = P(A_1A_5A_6)$$

$$= P(A_2A_3A_6) = P(A_2A_4A_6) = P(A_2A_5A_6) = \left(1-e^{-\lambda t}\right)^7 \; P(A_1A_3A_6) = P(A_1A_4A_6) = \left(1-e^{-\lambda t}\right)^8$$

$$P(A_1A_2A_3A_4) = P(A_2A_3A_4A_5) = P(A_3A_4A_5A_6) = \left(1-e^{-\lambda t}\right)^6$$

$$P(A_1A_2A_3A_5) = P(A_1A_2A_4A_5) = P(A_1A_3A_4A_5) = P(A_2A_3A_4A_6)$$

$$= P(A_2A_3A_5A_6) = P(A_2A_4A_5A_6) = \left(1-e^{-\lambda t}\right)^7$$

ISBN 978-602-8273-53-4
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$$P(A_1A_2A_3A_6)=P(A_1A_2A_4A_6)=P(A_1A_3A_5A_6)=P(A_1A_3A_4A_6)$$
$$=P(A_1A_3A_5A_6)=P(A_1A_4A_5A_6)=\left(1-e^{-\lambda t}\right)^8$$
$$P(A_1A_2A_3A_4A_5)=P(A_2A_3A_4A_5A_6)=\left(1-e^{-\lambda t}\right)^7$$
$$P(A_1A_2A_3A_4A_6)=P(A_1A_2A_3A_5A_6)=P(A_1A_2A_4A_5A_6)=P(A_1A_3A_4A_5A_6)=\left(1-e^{-\lambda t}\right)^8$$
$$P(A_1A_2A_3A_4A_5A_6)=\left(1-e^{-\lambda t}\right)^8$$
The system failure probability P(t) and reliability system R(t) are $P\left(A_1\cup A_2\cup A_3\cup A_4\cup A_5\cup A_6\right)$
$$=6\left(1-e^{-\lambda t}\right)^3-5\left(1-e^{-\lambda t}\right)^4-4\left(1-e^{-\lambda t}\right)^5-6\left(1-e^{-\lambda t}\right)^6+4\left(1-e^{-\lambda t}\right)^5+$$
$$6\left(1-e^{-\lambda t}\right)^6+8\left(1-e^{-\lambda t}\right)^7+2\left(1-e^{-\lambda t}\right)^8-3\left(1-e^{-\lambda t}\right)^6-6\left(1-e^{-\lambda t}\right)^7-$$
$$6\left(1-e^{-\lambda t}\right)^8+2\left(1-e^{-\lambda t}\right)^7+4\left(1-e^{-\lambda t}\right)^8-\left(1-e^{-\lambda t}\right)^8$$
$$R(t)=1-6\left(1-e^{-\lambda t}\right)^3+5\left(1-e^{-\lambda t}\right)^4+4\left(1-e^{-\lambda t}\right)^5+6\left(1-e^{-\lambda t}\right)^6-4\left(1-e^{-\lambda t}\right)^5-$$
$$6\left(1-e^{-\lambda t}\right)^6-8\left(1-e^{-\lambda t}\right)^7-2\left(1-e^{-\lambda t}\right)^8+3\left(1-e^{-\lambda t}\right)^6+6\left(1-e^{-\lambda t}\right)^7+$$
$$6\left(1-e^{-\lambda t}\right)^8-2\left(1-e^{-\lambda t}\right)^7-4\left(1-e^{-\lambda t}\right)^8+\left(1-e^{-\lambda t}\right)^8$$
(3) $n=8$, $k=2$

According to the definition and the above assumptions, the system has seven kinds of failure circumstances:
$$A_1(B_1B_2),A_2(B_2B_3),A_3(B_3B_4),A_4(B_4B_5),A_5(B_5B_6),A_6(B_6B_7),A_7(B_7B_8)$$
$$P(A_1)=P(A_2)=P(A_3)=P(A_4)=P(A_5)=P(A_6)=P(A_7)=\left(1-e^{-\lambda t}\right)^2$$
$$P(A_1A_2)=P(A_2A_3)=P(A_3A_4)=P(A_4A_5)=P(A_5A_6)=P(A_6A_7)=\left(1-e^{-\lambda t}\right)^3$$
$$P(A_1A_3)=P(A_1A_4)=P(A_1A_5)=P(A_1A_6)=P(A_1A_7)=P(A_2A_4)=P(A_2A_5)=$$
$$P(A_2A_6)=P(A_2A_7)=P(A_3A_5)=P(A_3A_6)=P(A_3A_7)=P(A_4A_6)=P(A_4A_7)=$$
$$P(A_5A_7)=\left(1-e^{-\lambda t}\right)^4$$
$$P(A_1A_2A_3)=P(A_2A_3A_4)=P(A_3A_4A_5)=P(A_4A_5A_6)=P(A_5A_6A_7)=\left(1-e^{-\lambda t}\right)^4$$
$$P(A_1A_2A_4)=P(A_1A_2A_5)=P(A_1A_2A_6)=P(A_1A_2A_7)=P(A_1A_3A_4)$$
$$=P(A_1A_4A_5)=P(A_1A_5A_6)=P(A_1A_6A_7)=P(A_2A_3A_5)=P(A_2A_3A_6)$$
$$=P(A_2A_3A_7)=P(A_2A_4A_5)=P(A_2A_5A_6)=P(A_2A_6A_7)=P(A_3A_4A_6)$$
$$=P(A_3A_4A_7)=P(A_3A_5A_6)=P(A_3A_6A_7)=P(A_4A_5A_7)=P(A_4A_6A_7)=\left(1-e^{-\lambda t}\right)^5$$
$$P(A_1A_3A_5)=P(A_1A_3A_6)=P(A_1A_3A_7)=P(A_1A_4A_6)=P(A_1A_4A_7)$$
$$=P(A_1A_5A_7)=P(A_2A_4A_6)=P(A_2A_4A_7)=P(A_2A_5A_7)=P(A_3A_5A_7)=\left(1-e^{-\lambda t}\right)^6$$
$$P(A_1A_2A_3A_4)=P(A_2A_3A_4A_5)=P(A_3A_4A_5A_6)=P(A_4A_5A_6A_7)=\left(1-e^{-\lambda t}\right)^5$$

$$P\left(A_1A_2A_3A_5\right)=P\left(A_1A_2A_3A_6\right)=P\left(A_1A_2A_3A_7\right)=P\left(A_1A_2A_4A_5\right)=P\left(A_1A_2A_5A_6\right)=$$
$$P\left(A_1A_2A_6A_7\right)=P\left(A_1A_3A_4A_5\right)=P\left(A_1A_4A_5A_6\right)=P\left(A_1A_5A_6A_7\right)=P\left(A_2A_3A_4A_6\right)=$$
$$P\left(A_2A_3A_4A_7\right)=P\left(A_2A_3A_5A_6\right)=P\left(A_2A_3A_6A_7\right)=P\left(A_2A_4A_5A_6\right)=P\left(A_2A_5A_6A_7\right)=$$
$$P\left(A_3A_4A_5A_7\right)=P\left(A_3A_4A_6A_7\right)=P\left(A_3A_5A_6A_7\right)=\left(1-e^{-\lambda t}\right)^6$$
$$P\left(A_1A_2A_4A_6\right)=P\left(A_1A_2A_4A_7\right)=P\left(A_1A_2A_5A_7\right)=P\left(A_1A_3A_4A_6\right)=$$
$$P\left(A_1A_3A_4A_7\right)=P\left(A_1A_3A_5A_6\right)=P\left(A_1A_3A_6A_7\right)=P\left(A_1A_4A_5A_7\right)=$$
$$P\left(A_1A_4A_6A_7\right)=P\left(A_2A_3A_5A_7\right)=P\left(A_2A_4A_5A_7\right)=P\left(A_2A_4A_6A_7\right)=\left(1-e^{-\lambda t}\right)^7$$
$$P\left(A_1A_3A_5A_7\right)=\left(1-e^{-\lambda t}\right)^8 \; P\left(A_1A_2A_3A_4A_5\right)=P\left(A_2A_3A_4A_5A_6\right)=P\left(A_3A_4A_5A_6A_7\right)=\left(1-e^{-\lambda t}\right)^6$$
$$P\left(A_1A_2A_3A_4A_6\right)=P\left(A_1A_2A_3A_4A_7\right)=P\left(A_1A_2A_3A_5A_6\right)=P\left(A_1A_2A_3A_6A_7\right)$$
$$=P\left(A_1A_2A_4A_5A_6\right)=P\left(A_1A_2A_5A_6A_7\right)=P\left(A_1A_3A_4A_5A_6\right)=P\left(A_1A_4A_5A_6A_7\right)$$
$$=P\left(A_2A_3A_4A_5A_7\right)=P\left(A_2A_3A_4A_6A_7\right)=P\left(A_2A_3A_5A_6A_7\right)=P\left(A_2A_2A_5A_6A_7\right)=\left(1-e^{-\lambda t}\right)^7$$
$$P\left(A_1A_2A_3A_5A_7\right)=P\left(A_1A_2A_4A_5A_7\right)=P\left(A_1A_2A_4A_6A_7\right)=P\left(A_1A_3A_4A_5A_7\right)$$
$$=P\left(A_1A_3A_4A_6A_7\right)=P\left(A_1A_3A_5A_6A_7\right)=\left(1-e^{-\lambda t}\right)^8$$
$$P\left(A_1A_2A_3A_4A_5A_7\right)=P\left(A_1A_2A_3A_4A_6A_7\right)=P\left(A_1A_2A_3A_5A_6A_7\right)=P\left(A_1A_2A_4A_5A_6A_7\right)$$
$$=P\left(A_1A_3A_4A_5A_6A_7\right)=\left(1-e^{-\lambda t}\right)^8$$
$$P\left(A_1A_2A_3A_4A_5A_6A_7\right)=\left(1-e^{-\lambda t}\right)^8$$

The system failure probability P(t) and reliability system R(t) are

$$P\left(A_1\cup A_2\cup A_3\cup A_4\cup A_5\cup A_6\cup A_7\right)$$
$$=7\left(1-e^{-\lambda t}\right)^2-6\left(1-e^{-\lambda t}\right)^3-16\left(1-e^{-\lambda t}\right)^4+5\left(1-e^{-\lambda t}\right)^4+20\left(1-e^{-\lambda t}\right)^5+$$
$$10\left(1-e^{-\lambda t}\right)^6-4\left(1-e^{-\lambda t}\right)^5-18\left(1-e^{-\lambda t}\right)^6-12\left(1-e^{-\lambda t}\right)^7-\left(1-e^{-\lambda t}\right)^8+$$
$$3\left(1-e^{-\lambda t}\right)^6+12\left(1-e^{-\lambda t}\right)^7+6\left(1-e^{-\lambda t}\right)^8-2\left(1-e^{-\lambda t}\right)^7-5\left(1-e^{-\lambda t}\right)^8+\left(1-e^{-\lambda t}\right)^8$$
$$=7\left(1-e^{-\lambda t}\right)^2-6\left(1-e^{-\lambda t}\right)^3-11\left(1-e^{-\lambda t}\right)^4+16\left(1-e^{-\lambda t}\right)^5-5\left(1-e^{-\lambda t}\right)^6-$$
$$2\left(1-e^{-\lambda t}\right)^7+\left(1-e^{-\lambda t}\right)^8$$
$$R(t)=1-7\left(1-e^{-\lambda t}\right)^2+6\left(1-e^{-\lambda t}\right)^3+11\left(1-e^{-\lambda t}\right)^4-16\left(1-e^{-\lambda t}\right)^5+5\left(1-e^{-\lambda t}\right)^6+$$
$$2\left(1-e^{-\lambda t}\right)^7-\left(1-e^{-\lambda t}\right)^8$$

4. Reliability Analysis of General Circumstances

(1) When $k<n\le 2k$

$$R(t)=1-P(A_1\cup A_2\cup A_3\cup\cdots\cup A_{n-k+1})=1-[(n-k+1)(1-e^{-\lambda t})^k$$
$$-\sum_{i=0}^{n-k-1}C_i^0(n-k-i)(1-e^{-\lambda t})^{k+1+i}+\sum_{i=1}^{n-k-1}C_i^1(n-k-i)(1-e^{-\lambda t})^{k+1+i}+\cdots+$$

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$$+(-1)^{m-1}\sum_{i=m-2}^{n-k-1}C_i^{m-2}(n-k-i)(1-e^{-\lambda t})^{k+1+i}+\cdots+(-1)^{n-2}C_{n-k-1}^{1}(1-e^{-\lambda t})^{n}+(-1)^{n-1}(1-e^{-\lambda t})^{n}]\ (2)$$

when $2k < n \le 3k$

$$R(t)=1-P(A_1\cup A_2\cup A_3\cup\cdots\cup A_{n-k+1})=1-[(n-k+1)(1-e^{-\lambda t})^{k}$$

$$-\sum_{i=0}^{k-2}C_i^0(n-k-i)(1-e^{-\lambda t})^{k+1+i}-\sum_{i=k-1}^{n-k-1}C_i^0(n-k-i)(1-e^{-\lambda t})^{2k}+\sum_{i=1}^{n-k-1}C_i^1(n-k-i)(1-e^{-\lambda t})^{k+1+i}$$

(3) when $3k < n \le 4k$

$$R(t)=1-P(A_1\cup A_2\cup A_3\cup\cdots\cup A_{n-k+1})=1-[(n-k+1)(1-e^{-\lambda t})^{k}$$

$$-\sum_{i=0}^{k-2}C_i^0(n-k-i)(1-e^{-\lambda t})^{k+1+i}-\sum_{i=k-1}^{n-k-1}C_i^0(n-k-i)(1-e^{-\lambda t})^{2k}+$$

$$\sum_{i=1}^{2k-2}C_i^1(n-k-i)(1-e^{-\lambda t})^{k+1+i}+\sum_{i=2k-1}^{n-k-1}C_i^1(n-k-i)(1-e^{-\lambda t})^{3k}-\sum_{i=2}^{n-k-1}C_i^2(n-k-i)(1-e^{-\lambda t})^{k+1+i}$$

$$+\cdots+(-1)^{m-1}\sum_{i=m-2}^{n-k-1}C_i^{m-2}(n-k-i)(1-e^{-\lambda t})^{k+1+i}+\cdots+(-1)^{n-2}C_{n-k-1}^{1}(1-e^{-\lambda t})^{n}+(-1)^{n-1}(1-e^{-\lambda t})^{n}]$$

Proof: （1） $P(A_iA_j)(i\neq j; i=1,2,\cdots,n-k; j=2,3,4,\cdots,n-k+1)$

Tabel 2. 1 Probability Calculation A_iA_j

A_iA_j	Total number of situations	Failed number of component	Probabilities (P)
$A_iA_{i+1}(i=1,2,\cdots,n-k)$	$n-k$	$k+1$	$=(n-k)(1-e^{-\lambda t})^{k+1}$
$A_iA_{i+2}(i=1,2,\cdots,n-k-1)$	$n-k-1$	$k+2$	$=(n-k-1)(1-e^{-\lambda t})^{k+2}$
$A_iA_{i+3}(i=1,2,\cdots,n-k-2)$	$n-k-2$	$k+3$	$=(n-k-2)(1-e^{-\lambda t})^{k+3}$
$\cdots$	$\cdots$	$\cdots$	$\cdots$
$A_iA_{i+n-k-1}(i=1,2)$	2	$n-1$	$=2(1-e^{-\lambda t})^{n-1}$
A_1A_{n-k+1}	1	n	$=(1-e^{-\lambda t})^{n}$

$P(A_iA_jA_l)(i\neq j\neq l; i=1,2,\cdots,n-k-1; j=2,3,4,\cdots,n-k; l=3,4,\cdots,n-k+1)$ calculation tabel 2.2

By this analogy

$$P\left(A_{i_1}A_{i_2}\ldots A_{i_m}\right)=\sum_{i=m-2}^{n-k+1}C_i^{m-2}\left(n-k-i\right)\left(1-e^{-\lambda t}\right)^{k+1+i}$$

$p(t)$ and $R(t)$ are

$$p = P(A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_{n-k+1})$$

$$= \sum_{i=1}^{n-k+1} P(A_i) - \sum_{i=1}^{n-k} \sum_{j=2,j>i}^{n-k+1} P(A_i A_j) + \sum_{i=1}^{n-k-1} \sum_{j=2,j>i}^{n-k} \sum_{l=3,l>j}^{n-k+1} P(A_i A_j A_l) + \cdots + (-1)^{n-k} P(A_1 A_2 A_3 \cdots A_{n-k} A_{n-k+1})$$

$$= (n-k+1)(1-e^{-\lambda t})^k - \sum_{i=0}^{n-k-1} C_i^0 (n-k-i)(1-e^{-\lambda t})^{k+1+i} + \sum_{i=1}^{n-k-1} C_i^1 (n-k-i)(1-e^{-\lambda t})^{k+1+i} + \cdots$$

$$+(-1)^{m-1} \sum_{i=m-2}^{n-k-1} C_i^{m-2} (n-k-i)(1-e^{-\lambda t})^{k+1+i} + \cdots + (-1)^{n-2} C_{n-k-1}^1 (1-e^{-\lambda t})^n + (-1)^{n-1} (1-e^{-\lambda t})^n$$

$$R(t) = 1 - P(A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_{n-k+1}) = 1 - [(n-k+1)(1-e^{-\lambda t})^k$$

$$- \sum_{i=0}^{n-k-1} C_i^0 (n-k-i)(1-e^{-\lambda t})^{k+1+i} + \sum_{i=1}^{n-k-1} C_i^1 (n-k-i)(1-e^{-\lambda t})^{k+1+i} + \cdots +$$

$$+(-1)^{m-1} \sum_{i=m-2}^{n-k-1} C_i^{m-2} (n-k-i)(1-e^{-\lambda t})^{k+1+i} + \cdots + (-1)^{n-2} C_{n-k-1}^1 (1-e^{-\lambda t})^n + (-1)^{n-1} (1-e^{-\lambda t})^n]$$

Tabel 2.2 Probability Calculation $A_i A_j A_l$

$A_i A_j A_l$	Total number of situations	Failed number of component	Probabilities (P)
$A_i A_{i+1} A_{i+2} (i = 1,2,\cdots,n-k-1)$	$n-k-1$	$k+2$	$= C_1^1 (n-k-1)(1-e^{-\lambda t})^{k+2}$
$A_i A_{i+2} A_{i+3}$或$A_i A_{i+1} A_{i+3}$ $(i = 1,2,\cdots,n-k-1)$	$n-k-2$	$k+3$	$= C_2^1 (n-k-2)(1-e^{-\lambda t})^{k+3}$
$A_i A_{i+3} A_{i+4}$或$A_i A_{i+1} A_{i+4}$或$A_i A_{i+2} A_{i+4}$ $(i = 1,2,\cdots,n-k-3)$	$n-k-3$	$k+3$	$= C_3^1 (n-k-3)(1-e^{-\lambda t})^{k+4}$
$\cdots$	$\cdots$	$\cdots$	$\cdots$
$A_i A_j A_{i+n-k-1} (i = 1,2;$ $j = 2,3,\cdots,n-k-1; j > i)$	2	$n-1$	$= 2C_{n-k-2}^1 (1-e^{-\lambda t})^{n-1}$
$A_1 A_j A_{n-k+1} (j = 2,3,\cdots,n-k)$	1	n	$= C_{n-k-1}^1 (1-e^{-\lambda t})^n$

（2） $P(A_i A_j)(i \neq j; i = 1,2,\cdots,n-k; j = 2,3,4,\cdots,n-k+1)$ Calculation Tabel 2.3

Tabel 2.3 Probability Calculation $A_i A_j$

$A_i A_j$	Total number of situations	Failed number of component	Probabilities (P)
$A_i A_{i+1} (i = 1,2,\cdots,n-k)$	$n-k$	$k+1$	$= (n-k)(1-e^{-\lambda t})^{k+1}$
$A_i A_{i+2} (i = 1,2,\cdots,n-k-1)$	$n-k-1$	$k+2$	$= (n-k-1)(1-e^{-\lambda t})^{k+2}$
$A_i A_{i+3} (i = 1,2,\cdots,n-k-2)$	$n-k-2$	$k+3$	$= (n-k-2)(1-e^{-\lambda t})^{k+3}$
$\cdots$	$\cdots$	$\cdots$	$\cdots$

$A_i A_{i+k-1} (i = 1,2,\cdots,n-2k+2)$	$n-2k+2$	$2k-1$	$= (n-2k+2)(1-e^{-\lambda t})^{2k-1}$
$A_i A_{i+k} (i = 1,2,\cdots,n-2k+1)$	$n-2k+1$	$2k$	$= (n-2k+1)(1-e^{-\lambda t})^{2k}$
$A_i A_{i+k+1} (i = 1,2,\cdots,n-2k)$	$n-2k$	$2k$	$= (n-2k)(1-e^{-\lambda t})^{2k}$
$\cdots$	$\cdots$	$\cdots$	$\cdots$
$A_1 A_{n-k+1}$	1	$2k$	$= (1-e^{-\lambda t})^{2k}$

$P(A_i A_j A_l)(i \neq j \neq l; i = 1,2,\cdots,n-k-1; j = 2,3,4,\cdots,n-k; l = 3,4,\cdots,n-k+1)$

（Note：this probability is the same with $k < n \leq 2k$） Calculation tabel2.4.

$p(t)$ and $R(t)$ are respectively:

$$p = P(A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_{n-k+1}) = \sum_{i=1}^{n-k+1} P(A_i) - \sum_{i=1}^{n-k} \sum_{j=2, j>i}^{n-k+1} P(A_i A_j) + \sum_{i=1}^{n-k-1} \sum_{j=2, j>i}^{n-k} \sum_{l=3, l>j}^{n-k+1} P(A_i A_j A_l)$$

$$+ \cdots + (-1)^{n-k} P(A_1 A_2 A_3 \cdots A_{n-k} A_{n-k+1})$$

$$= (n-k+1)(1-e^{-\lambda t})^k - \sum_{i=0}^{k-2} C_i^0 (n-k-i)(1-e^{-\lambda t})^{k+1+i} - \sum_{i=k-1}^{n-k-1} C_i^0 (n-k-i)(1-e^{-\lambda t})^{2k} +$$

$$\sum_{i=1}^{n-k-1} C_i^1 (n-k-i)(1-e^{-\lambda t})^{k+1+i} + \cdots + (-1)^{m-1} \sum_{i=m-2}^{n-k-1} C_i^{m-2} (n-k-i)(1-e^{-\lambda t})^{k+1+i}$$

$$+ \cdots + (-1)^{n-2} C_{n-k-1}^1 (1-e^{-\lambda t})^n + (-1)^{n-1} (1-e^{-\lambda t})^n$$

$$R(t) = 1 - P(A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_{n-k+1}) = 1 - [(n-k+1)(1-e^{-\lambda t})^k$$

$$- \sum_{i=0}^{k-2} C_i^0 (n-k-i)(1-e^{-\lambda t})^{k+1+i} - \sum_{i=k-1}^{n-k-1} C_i^0 (n-k-i)(1-e^{-\lambda t})^{2k} + \sum_{i=1}^{n-k-1} C_i^1 (n-k-i)(1-e^{-\lambda t})^{k+1+i}$$

$$+ \cdots + (-1)^{m-1} \sum_{i=m-2}^{n-k-1} C_i^{m-2} (n-k-i)(1-e^{-\lambda t})^{k+1+i} + \cdots + (-1)^{n-2} C_{n-k-1}^1 (1-e^{-\lambda t})^n + (-1)^{n-1} (1-e^{-\lambda t})^n]$$

tabel.4 Probability Calculation $A_i A_j A_l$

$A_i A_j A_l$	Total number of situations	Failed number of component	Probabilities (P)
$A_i A_{i+1} A_{i+2} (i = 1,2,\cdots,n-k-1)$	$n-k-1$	$k+2$	$= C_1^1 (n-k-1)(1-e^{-\lambda t})^{k+2}$
$A_i A_{i+2} A_{i+3}$或$A_i A_{i+1} A_{i+3}$ $(i = 1,2,\cdots,n-k-1)$	$n-k-2$	$k+3$	$= C_2^1 (n-k-2)(1-e^{-\lambda t})^{k+3}$
$A_i A_{i+3} A_{i+4}$或$A_i A_{i+1} A_{i+4}$或$A_i A_{i+2} A_{i+4}$ $(i = 1,2,\cdots,n-k-3)$	$n-k-3$	$k+4$	$= C_3^1 (n-k-3)(1-e^{-\lambda t})^{k+4}$
$\cdots$	$\cdots$	$\cdots$	$\cdots$
$A_i A_j A_{i+n-k-1} (i = 1,2;$ $j = 2,3,\cdots,n-k-1; j > i)$	2	$n-1$	$= 2C_{n-k-2}^1 (1-e^{-\lambda t})^{n-1}$

ISBN 978-602-8273-53-4
www.isc.unwahas.ac.id

$A_1A_jA_{n-k+1}(j=2,3,\cdots,n-k)$	1	n	$=C_{n-k-1}^{1}(1-e^{-\lambda t})^{n}$

(3) When $3k<n\le 4k$, $P(A_iA_j)(i\ne j;i=1,2,\cdots,n-k;j=2,3,4,\cdots,n-k+1)$ and same with when $2k<n\le 3k$, tabel 2.3.
$P(A_iA_jA_l)(i\ne j\ne l;i=1,2,\cdots,n-k-1;j=2,3,4,\cdots,n-k;l=3,4,\cdots,n-k+1)$, 计Calculation Tabel 2.5. $p(t)$ and $R(t)$ are respectively: :

$$p=P(A_1\cup A_2\cup A_3\cup\cdots\cup A_{n-k+1})=\sum_{i=1}^{n-k+1}P(A_i)-\sum_{i=1}^{n-k}\sum_{j=2,j>i}^{n-k+1}P(A_iA_j)+\sum_{i=1}^{n-k-1}\sum_{j=2,j>i}^{n-k}\sum_{l=3,l>j}^{n-k+1}P(A_iA_jA_l)$$
$$+\cdots+(-1)^{n-k}P(A_1A_2A_3\cdots A_{n-k}A_{n-k+1})$$
$$=(n-k+1)(1-e^{-\lambda t})^{k}-\sum_{i=0}^{k-2}C_i^0(n-k-i)(1-e^{-\lambda t})^{k+1+i}-\sum_{i=k-1}^{n-k-1}C_i^0(n-k-i)(1-e^{-\lambda t})^{2k}$$
$$+\sum_{i=1}^{2k-2}C_i^1(n-k-i)(1-e^{-\lambda t})^{k+1+i}+\sum_{i=2k-1}^{n-k-1}C_i^1(n-k-i)(1-e^{-\lambda t})^{3k}-\sum_{i=2}^{n-k-1}C_i^2(n-k-i)(1-e^{-\lambda t})^{k+1+i}$$
$$\cdots+(-1)^{m-1}\sum_{i=m-2}^{n-k-1}C_i^{m-2}(n-k-i)(1-e^{-\lambda t})^{k+1+i}+\cdots+(-1)^{n-2}C_{n-k-1}^{1}(1-e^{-\lambda t})^{n}+(-1)^{n-1}(1-e^{-\lambda t})^{n}$$
$$R(t)=1-P(A_1\cup A_2\cup A_3\cup\cdots\cup A_{n-k+1})=1-[(n-k+1)(1-e^{-\lambda t})^{k}$$
$$-\sum_{i=0}^{k-2}C_i^0(n-k-i)(1-e^{-\lambda t})^{k+1+i}-\sum_{i=k-1}^{n-k-1}C_i^0(n-k-i)(1-e^{-\lambda t})^{2k}+$$
$$\sum_{i=1}^{2k-2}C_i^1(n-k-i)(1-e^{-\lambda t})^{k+1+i}+\sum_{i=2k-1}^{n-k-1}C_i^1(n-k-i)(1-e^{-\lambda t})^{3k}-\sum_{i=2}^{n-k-1}C_i^2(n-k-i)(1-e^{-\lambda t})^{k+1+i}$$

Tabel 2.5 Probability Calculation $A_iA_jA_l$

$A_iA_jA_l$	Total situations	Filed component	Probability (P)
$A_iA_{i+1}A_{i+2}(i=1,2,\cdots,n-k-1)$	$n-k-1$	$k+2$	$=C_1^1(n-k-1)(1-e^{-\lambda t})^{k+2}$
$A_iA_{i+2}A_{i+3}$或$A_iA_{i+1}A_{i+3}$ $(i=1,2,\cdots,n-k-1)$	$n-k-2$	$k+3$	$=C_2^1(n-k-2)(1-e^{-\lambda t})^{k+3}$
$A_iA_{i+3}A_{i+4}$或$A_iA_{i+1}A_{i+4}$或$A_iA_{i+2}A_{i+4}$ $(i=1,2,\cdots,n-k-3)$	$n-k-3$	$k+4$	$=C_3^1(n-k-3)(1-e^{-\lambda t})^{k+4}$
…	…	…	…
$A_iA_jA_{i+2k-1}(i=1,2,\cdots n-3k+2;$ $j=2,3,\cdots,n-3k+1;j>i)$	$n-3k+2$	$3k+1$	$=C_{2k-2}^{1}(n-3k+2)(1-e^{-\lambda t})^{3k+1}$
$A_iA_jA_{i+2k}(i=1,2,\cdots n-3k+1;$ $j=2,3,\cdots,n-3k;j>i)$	$n-3k+1$	$3k$	$=C_{2k-1}^{1}(n-3k+1)(1-e^{-\lambda t})^{3k}$
$A_iA_jA_{i+2k+1}(i=1,2,\cdots n-3k;$ $j=2,3,\cdots,n-3k-1;j>i)$	$n-3k$	$3k$	$=C_{2k}^{1}(n-3k)(1-e^{-\lambda t})^{3k}$
…	…	…	…

$A_i A_j A_{i+n-k-1}(i=1,2;$ $j=2,3,\cdots,n-k-1)$	2	$3k$	$=2C_{n-k-2}^{1}(1-e^{-\lambda t})^{3k}$
$A_1 A_j A_{n-k+1}(j=2,3,\cdots,n-k)$	1	$3k$	$=C_{n-k-1}^{1}(1-e^{-\lambda t})^{3k}$

5. Conclusion

It can be deduced from the above three classifications: when $jk < n \le (j+1)k(j \ge 2)$, $p(t)$ and $R(t)$ are respectively:

$$p = P(A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_{n-k+1}) = \sum_{i=1}^{n-k+1} P(A_i) - \sum_{i=1}^{n-k}\sum_{j=2,j>i}^{n-k+1} P(A_i A_j) + \sum_{i=1}^{n-k-1}\sum_{j=2,j>i}^{n-k}\sum_{l=3,l>j}^{n-k+1} P(A_i A_j A_l)$$

$$+\cdots+(-1)^{n-k} P(A_1 A_2 A_3 \cdots A_{n-k} A_{n-k+1})$$

$$=(n-k+1)(1-e^{-\lambda t})^{k} - \sum_{i=0}^{k-2} C_i^0 (n-k-i)(1-e^{-\lambda t})^{k+1+i} - \sum_{i=k-1}^{n-k-1} C_i^0 (n-k-i)(1-e^{-\lambda t})^{2k}$$

$$+\sum_{i=1}^{2k-2} C_i^1 (n-k-i)(1-e^{-\lambda t})^{k+1+i} + \sum_{i=2k-1}^{n-k-1} C_i^1 (n-k-i)(1-e^{-\lambda t})^{3k} -$$

$$\sum_{i=2}^{3k-2} C_i^2 (n-k-i)(1-e^{-\lambda t})^{k+1+i} - \sum_{i=3k-1}^{n-k-1} C_i^2 (n-k-i)(1-e^{-\lambda t})^{4k} \cdots +$$

$$(-1)^{j-1}\sum_{i=j-2}^{(j-1)k-2} C_i^{j-2} (n-k-i)(1-e^{-\lambda t})^{k+1+i} + (-1)^{j-1}\sum_{i=(j-1)k-1}^{n-k-1} C_i^{j-2} (n-k-i)(1-e^{-\lambda t})^{jk} +$$

$$(-1)^{m-1}\sum_{i=m-2,m>j}^{n-k-1} C_i^{m-2} (n-k-i)(1-e^{-\lambda t})^{k+1+i} + \cdots + (-1)^{n-2} C_{n-k-1}^{1} (1-e^{-\lambda t})^{n} + (-1)^{n-1}(1-e^{-\lambda t})^{n}$$

$$R(t) = 1 - P(A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_{n-k+1}) = 1 - [(n-k+1)(1-e^{-\lambda t})^{k}$$

$$-\sum_{i=0}^{k-2} C_i^0 (n-k-i)(1-e^{-\lambda t})^{k+1+i} - \sum_{i=k-1}^{n-k-1} C_i^0 (n-k-i)(1-e^{-\lambda t})^{2k}$$

$$+\sum_{i=1}^{2k-2} C_i^1 (n-k-i)(1-e^{-\lambda t})^{k+1+i} + \sum_{i=2k-1}^{n-k-1} C_i^1 (n-k-i)(1-e^{-\lambda t})^{3k} -$$

$$\sum_{i=2}^{3k-2} C_i^2 (n-k-i)(1-e^{-\lambda t})^{k+1+i} - \sum_{i=3k-1}^{n-k-1} C_i^2 (n-k-i)(1-e^{-\lambda t})^{4k} \cdots +$$

$$(-1)^{j-1}\sum_{i=j-2}^{(j-1)k-2} C_i^{j-2} (n-k-i)(1-e^{-\lambda t})^{k+1+i} + (-1)^{j-1}\sum_{i=(j-1)k-1}^{n-k-1} C_i^{j-2} (n-k-i)(1-e^{-\lambda t})^{jk} +$$

$$(-1)^{m-1}\sum_{i=m-2,m>j}^{n-k-1} C_i^{m-2} (n-k-i)(1-e^{-\lambda t})^{k+1+i} + \cdots + (-1)^{n-2} C_{n-k-1}^{1} (1-e^{-\lambda t})^{n} + (-1)^{n-1}(1-e^{-\lambda t})^{n}]$$

The above formulas serve as references for the calculation of the reliability of linear consecutive $k-out-of-n$ systems of different components and different types in engineering, which can be done only by putting the corresponding parameters. The reliability calculation formula of other distribution parameters can also be similarly obtained by following this classification.

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