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Linear Consecutive 3/7:F Repairable System with Priority in Repair

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Abstract-A Linear consecutive 3-out-of-7:F repairable system with one repairman is studied in this paper. When there are more than one failed component, priorities are assigned to the failed components.Assuming that system has only one repairman, both the working time and the repair time of a component are exponentially distributed. The rapirman assigned different priorities to the failed components. Through the analysis of the system and get some important reliability index of the system by using the general markov process and the tool of the Laplace transformation.

1. Introduction

A linear consecutive $k-out-of-n:F$ system consists of a sequence of n ordered components such that the system is failed if and only if at least k consecutive components in the system are failed. A telecommunications system with n relay stations can be modeled as a linear consecutive $k-out-of-n:F$ system. Suppose that the stations numbered consecutively from 1 to n are lined up and the signal transmitted from each station is strong enough to reach the next k stations. The communication between the source and the destination will be interrupted if and only if at least k consecutive stations are failed. Another example of the consecutive- $k-out-of-n:F$ system model is an oil pipeline system with n pump stations. Each station is powerful enough to send oil as far as to the next k stations. If less than k consecutive stations are failed, the flow of oil will not be interrupted and the pipeline is considered working properly.

A circular consecutive $k-out-of-n:F$ system consists of n components arranging along a circular path. This system will fail if and only if at least k consecutive components in the system are failed. Components n and 1 are considered consecutive components in such a circular system. A closed recurring water supply system with n water pumps in a thermoelectric plant is a good example of this system. The water and steam expelled from a turbine can be pumped to a cooling tower through n water pumps. Because of the disparity of the water level, the cooled water can be sent back to the boiler to produce steam for the turbine again. For such a recurring water-cooling system, each pump must be powerful enough to pump water and steam to at least the next two consecutive pumps. If any two or more consecutive pumps in the system are failed, the system is failed. This system can be modeled as a circular consecutive $k-out-of-n:F$ system.

In this paper, we researched a linear consecutive $k-out-of-n$ repairable system with priority in repair. The repairman assigned different priorities to the failed components. Through the analysis of the system and get some important reliability index of the system by using the general markov process and the tool of the Laplace transformation.

2. Assumptions

The discussion of this chapter is based the following hypotheses:

(1) If $t=0$, every component is in good condition, and the component can be recovered as new.

(2) Set the distribution functions of the working time and repair time of the i component in the system are separately

$$F(t)=1-e^{-\lambda t}\text{,}\quad G(t)=1-e^{-\mu t}\text{,}\quad t\geq 0, i=1,2,3,\ldots,n$$

(3) Set $X_i, Y_i, i=1,2,3,\ldots,n$ as the independent random variables

3. General Model: linear consecutive k /n:F repairable system

Based on the model assumptions, let $N(t)$ be the state of the system at time t, defined by

$N(t)=0$, if at time *t*, all components work and the system works,

$N(t)=-1$, if at time *t*, one component fails and the system works,

$N(t)=-2$, if at time *t*, two component fails and the system works,

⋮

$N(t)=-k$, if at time *t*, *k* component fails and the system works,

⋮

$N(t)=-d$, if at time *t*, d component fails and the system fails,

$N(t)=k$, if at time *t*, k component fails and the system fails,

$N(t)=k+1$, if at time *t*, k+1 component fails and the system fails,

⋮

$N(t)=d+1$, if at time t, d component fails and the system fails.

With $d=n-\left[\dfrac{n}{k}\right]$; $n\geq k$, $n,k\in N^{+}$,

state space $\Omega=\{0,-1,-2,\ldots,-d,k,k+1,\ldots,d+1\}$

the set of working states is $W=\{0,-1,-2,\ldots,-d\}$

the set of failed states is $F=\{k,k+1,\ldots,d+1\}$

By using the generalized transition probability shown earlier in this section and the concept of critical component, we have

$$P_{0j}(\Delta t)=\begin{cases} n\lambda\Delta t+0(\Delta t), & j=-1\\ 1-n\lambda\Delta t+0(\Delta t) & j=0\\ 0(\Delta t) & j\neq 0,-1\end{cases}$$

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$$P_{-ij}(\Delta t)=\begin{cases} iu(\Delta t)+0(\Delta t) & j=-(i-1) \\ (i+1)\dfrac{M_{-(i+1)}}{M_{-i}}\lambda\Delta t+0(\Delta t) & j=-(i+1) \\ \left[(n-i)-(n+1)\dfrac{M_{-(i+1)}}{M_{-i}}\right]\lambda\Delta t+0(\Delta t) & j=i+1 \\ 1-\left[(n-i)\lambda+iu\right]\lambda\Delta t+0(\Delta t) & j=-i \\ 0(\Delta t) & j\neq-(i-1),-(i+1),-i,i+1 \end{cases}$$

With $i=1,2,....,d$

$$P_{ij}(\Delta t)=\begin{cases} iu(\Delta t)+0(\Delta t) & j=-(i-1) \\ 1-iu(\Delta t)+0(\Delta t) & j=i \\ 0(\Delta t) & j\neq-(i-1),i \end{cases}$$

With $i=k,k+1,...,d+1$

4. Special model: linear consecutive 3 /7:F repairable system

We have a circular consecutive $k-out-of-n:F$ system. Based on the definitions given in the previous section, we know the state space of the circular consecutive $3-out-of-7:F$ system $J=\{0,-1,-2,-3,-4,-5,3,4,5,6\}$, the set of working states $W=\{0,-1,-2,-3,-4,-5\}$, and the set of failed states $F=\{3,4,5,6\}$. The transition rate matrix is:

$$A=\begin{pmatrix} -7\lambda & 7\lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \mu & -\mu-6\lambda & 6\lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2\mu & -2\mu-5\lambda & 30\lambda/7 & 0 & 0 & 5\lambda/7 & 0 & 0 & 0 \\ 0 & 0 & 3\mu & -3\mu-4\lambda & 20\lambda/7 & 0 & 0 & 8\lambda/7 & 0 & 0 \\ 0 & 0 & 0 & 4\mu & -4\mu-3\lambda & 2\lambda/7 & 0 & 0 & 19\lambda/7 & 0 \\ 0 & 0 & 0 & 0 & 5\mu & -5\mu-4\lambda & 0 & 0 & 0 & 4\lambda \\ 0 & 0 & 3\mu & 0 & 0 & 0 & -3\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & 4\mu & 0 & 0 & 0 & -4\mu & 0 & 0 \\ 0 & 0 & 0 & 0 & 5\mu & 0 & 0 & 0 & -5\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & 6\mu & 0 & 0 & 0 & -6\mu \end{pmatrix}$$

5. Reliability Indexes of the Linear consecutive 3 /7:F System

In the following, we study the reliability indexes of the consecutive $3-out-of-7:F$ system

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(1) System availability

$$A=\sum_{j\in W}\pi_j=\pi_0+\pi_{-1}+\pi_{-2}+\pi_{-3}+\pi_{-4}+\pi_{-5}$$

$$=\frac{-196\mu^4-1372\lambda\mu^3-4116\lambda^2\mu^2-9205\lambda^3\mu+5600\lambda^4}{1372\mu^3\lambda}\cdot$$

$$\frac{588\mu^5-4116\lambda\mu^4+12348\lambda^2\mu^3+24675\lambda^3\mu^2+24504\lambda^4\mu+8320\lambda^5}{4116\lambda\mu^4}$$

(2) System reliability

$$\left(p'_0(t),p'_{-1}(t),p'_{-2}(t),p'_{-3}(t),p'_{-4}(t),p'_{-5}(t)\right)$$

$$=\left(p_0(t),p_{-1}(t),p_{-2}(t),p_{-3}(t),p_{-4}(t),p_{-5}(t)\right)\begin{pmatrix}-7\lambda & 7\lambda & 0 & 0 & 0 & 0\\ \mu & -\mu-6\lambda & 6\lambda & 0 & 0 & 0\\ 0 & 2\mu & -2\mu-5\lambda & \frac{30}{7}\lambda & 0 & 0\\ 0 & 0 & 3\mu & -3\mu-4\lambda & \frac{20}{7}\lambda & 0\\ 0 & 0 & 0 & 4\mu & -4\mu-3\lambda & \frac{2}{7}\lambda\\ 0 & 0 & 0 & 0 & 5\mu & -5\mu-4\lambda\end{pmatrix}$$

We get

$$p'_0(t)=-7\lambda p_0(t)+\mu p_{-1}(t)$$

$$p'_{-1}(t)=7\lambda p_0(t)-(\mu+6\lambda)p_{-1}(t)+2\mu p_{-2}(t)$$

$$p'_{-2}(t)=6\lambda p_{-1}(t)-(2\mu+5\lambda)p_{-2}(t)+3\mu p_{-3}(t)$$

$$p'_{-3}(t)=\frac{30}{7}\lambda p_{-2}(t)-(3\mu+4\lambda)p_{-3}(t)+4\mu p_{-4}(t)$$

$$p'_{-4}(t)=\frac{20}{7}\lambda p_{-3}(t)-(4\mu+3\lambda)p_{-4}(t)+5\mu p_{-5}(t)$$

$$p'_{-5}(t)=\frac{2}{7}\lambda p_{-4}(t)-(5\mu+4\lambda)p_{-5}(t)$$

Where $Q_W(0)=(p_0(t),p_{-1}(t),...,p_{-5}(t))=(1,0,0,0,0,0)$

Using the Laplace Transform, we get

$$sp*_0(s)-1=-7\lambda p*_0(s)+\mu p*_{-1}(s)$$

$$sp*_{-1}(s)=7\lambda p*_0(s)-(\mu+6\lambda)p*_{-1}(s)+2\mu p*_{-2}(s)$$

$$sp*_{-2}(s)=6\lambda p*_{-1}(s)-(2\mu+5\lambda)p*_{-2}(s)+3\mu p*_{-3}(s)$$

$$sp*_{-3}(s)=\frac{30}{7}\lambda p*_{-2}(s)-(3\mu+4\lambda)p*_{-3}(s)+4\mu p*_{-4}(s)$$

$$sp*_{-4}(s)=\frac{20}{7}\lambda p*_{-3}(s)-(4\mu+3\lambda)p*_{-4}(s)+5\mu p*_{-5}(s)$$

$$sp*_{-5}(s)=\frac{2}{7}\lambda p*_{-4}(s)-(5\mu+4\lambda)p*_{-5}(s)$$

Simplifying these equations, we have

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$$p*_0(s)=\frac{1+\mu p*_{-1}(s)}{s+7\lambda}$$
$$p*_{-1}(s)=\frac{7\lambda+(s+7\lambda)2\mu p*_{-2}(s)}{s^2+(\mu+13\lambda)s+6\lambda^2}$$
$$p*_{-2}(s)=\frac{42\lambda^2+3\mu p*_{-3}(s)}{s^3+(3\mu+18\lambda)s^2+2\mu^2 s+(19\mu\lambda+71\lambda^2)s+72\mu\lambda^2+30\lambda^3}$$
$$p*_{-3}(s)=\frac{-1260\lambda^2(20s+100\mu+80\lambda)h_1}{(20s+100\mu+80\lambda)^2 g_1-f_1h_1(20s+100\mu+80\lambda)}$$
$$p*_{-4}(s)=\frac{-1260\lambda^2(20s+100\mu+80\lambda)}{(20s+100\mu+80\lambda)g_1-f_1h_1}$$
$$p*_{-5}(s)=\frac{-360\lambda^3(20s+100\mu+80\lambda)}{(20s+100\mu+80\lambda)(s+5\mu+4\lambda)g_1-(s+5\mu+4\lambda)f_1h_1}$$
where
$$f_1=(42\mu+154\lambda)s^3+(7+77\mu^2+595\mu\lambda+1001\lambda^2)s^2+(1519\mu\lambda^2+2198\lambda^3+42\mu^3+455\mu^2\lambda)s-$$
$$90\mu-1512\mu^2\lambda^2-1386\mu\lambda^3+840\lambda^4$$
$$g_1=28\mu s^3+(84\mu^2+504\mu\lambda)s^2+(56\mu^3+532\mu^2\lambda+1988\mu\lambda^2)s-2016\mu^2\lambda^2+840\mu\lambda^3$$
$$h_1=7s^2+(63\mu+49\lambda)s+140\mu^2+207\mu\lambda+84\lambda^2$$
The Laplace transform of the system's reliability $R(t)$ is given by
$$R*(s)=\sum_{i\in W}p*_i(s)=p*_0(s)+p*_{-1}(s)+p*_{-2}(s)+p*_{-3}(s)+p*_{-4}(s)+p*_{-5}(s)$$

(3) The system's mean time to first failure (MTTFF)

Since $(x_0,x_1,...,x_K)B=-Q_W(0)$
$$(x_0,x_1,x_2,x_3,x_4,x_5)\begin{pmatrix}-7\lambda & 7\lambda & 0 & 0 & 0 & 0\\ \mu & -\mu-6\lambda & 6\lambda & 0 & 0 & 0\\ 0 & 2\mu & -2\mu-5\lambda & \frac{30}{7}\lambda & 0 & 0\\ 0 & 0 & 3\mu & -3\mu-4\lambda & \frac{20}{7}\lambda & 0\\ 0 & 0 & 0 & 4\mu & -4\mu-3\lambda & \frac{2}{7}\lambda\\ 0 & 0 & 0 & 0 & 5\mu & -5\mu-4\lambda\end{pmatrix}=-Q_W(0)$$
We get
$$x_0=\frac{1+\mu x_1}{7\lambda}$$
$$x_1=\frac{1+2\mu x_2}{6\lambda}$$

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$$x_2 = \frac{\left(105\mu\lambda + 140\lambda^2 - 18\mu\right) f_2 + \left(18\mu f_2 + 1764000\mu^3\lambda^2 + 1411200\mu^2\lambda^3\right)}{\left(500\mu\lambda^2 + 700\lambda^3 - 90\mu\lambda\right) f_2}$$

$$x_3 = \frac{6 f_2 + 588000\mu^2\lambda^2 + 470400\mu\lambda^3}{\left(105\mu\lambda + 140\lambda^2 - 18\mu\right) f_2}$$

$$x_4 = \frac{4200\mu\lambda + 3360\lambda^2}{f_2}$$

$$x_5 = \frac{8400\lambda^2\mu + 6720\lambda^3}{\left(35\mu + 28\lambda\right) f_2}$$

Where

$f_2 = 67480\mu^3 + 109025\mu^2\lambda^2 + 186200\mu\lambda^3 + 82642\mu^2\lambda + 10584\mu\lambda^2 + 82320\lambda^4$ The system's mean time to first failure (MTTFF) is given by

$$MTTFF = x_0 + x_1 + x_2 + x_3 + x_4 + x_5$$

(4) The system's rate of occurrence of failure (ROCOF)

$$M = \lim_{t\to\infty} m(t) = \sum_{k\in W} \pi_k \sum_{j\in F} a_{kj} = \pi_W C e_F$$

$$= \pi_0 \left(a_{03} + a_{04} + a_{05} + a_{06}\right) + \pi_{-1}\left(a_{-13} + a_{-14} + a_{-15} + a_{1-6}\right) + \pi_{-2}\left(a_{-23} + a_{-24} + a_{-25} + a_{-26}\right)$$

$$+ \pi_{-3}\left(a_{-33} + a_{-34} + a_{-35} + a_{-36}\right) + \pi_{-4}\left(a_{-43} + a_{-44} + a_{-45} + a_{-46}\right) + \pi_{-5}\left(a_{-53} + a_{-54} + a_{-55} + a_{-56}\right)$$

$$= \sum_{i\in W} \pi_i \left(a_{i3} + a_{i4} + a_{i5} + a_{i6}\right)$$

(5) The system's MTBF/ MUT

The MTBF of the system is also called the mean up time(MUT). Furthermore, we can also obtain mean down-time(MDT) and the mean cycle time (MCT) of the system:

$$MUT = \frac{A}{M}$$

$$= \frac{\frac{-196\mu^4 - 1372\lambda\mu^3 - 4116\lambda^2\mu^2 - 9205\lambda^3\mu + 5600\lambda^4}{1372\mu^3\lambda}\left(\frac{588\mu^5 - 4116\lambda\mu^4 + 12348\lambda^2\mu^3 + 24675\lambda^3\mu^2 + 24504\lambda^4\mu + 8320\lambda^5}{4116\lambda\mu^4}\right)}{\sum_{i\in W}\pi_i\left(a_{i3} + a_{i4} + a_{i5} + a_{i6}\right)}$$

$$MDT = \frac{\overline{A}}{M}, \quad \overline{A} = 1 - A$$

$$= \frac{1 - \frac{-196\mu^4 - 1372\lambda\mu^3 - 4116\lambda^2\mu^2 - 9205\lambda^3\mu + 5600\lambda^4}{1372\mu^3\lambda}\left(\frac{588\mu^5 - 4116\lambda\mu^4 + 12348\lambda^2\mu^3 + 24675\lambda^3\mu^2 + 24504\lambda^4\mu + 8320\lambda^5}{4116\lambda\mu^4}\right)}{\sum_{i\in W}\pi_i\left(a_{i3} + a_{i4} + a_{i5} + a_{i6}\right)}$$

$$MCT = \frac{1}{M}$$

$$= \frac{1}{\sum_{i\in W}\pi_i\left(a_{i3} + a_{i4} + a_{i5} + a_{i6}\right)}.$$

6. Conclusion

In this paper, a linear consecutive $k - out - of - n : F$ repairable system with one repairman is analyzed. First of all, the generalized transition probability is introduced to handle a state with many cases. Secondly, the concept of critical component is defined which is used to determine which of the two or more failed components should be repaired first. Priorities are assigned to critical components for repair. In addition, equations for the system's reliability indexes such as availability, reliability, MTTFF, ROCOF and MTBF/MUT are developed when $n = 7$. Problems worth further investigations include other repairable consecutive- k-out-of-n:F models, the impact of repair on availability of practical consecutive-k-out-of-n:F systems in petrochemical industries, and optimal repair/replacement strategies for operation/maintenance cost minimization in industrial environment.

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