ERROR COMPENSATION OF A GRINDING MACHINE TOOL SPINDLE BY OPTIMIZATION DESIGN

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Abstrak

Dalam makalah ini, kesalahan radial dari spindle mesin gerinda CNC presisi tinggi yang diakibatkan oleh pengaruh gaya unbalance telah diamati. Poros spindle dianggap sebagai rotor fleksibel yang didukung oleh dua buah angular contact ball bearing. Metode elemen hingga (FEM) telah diadopsi untuk mendapatkan persamaan gerak spindle. Dalam studi ini, pertama, frekuensi natural, kecepatan kritis dan amplitudo respon unbalance ditentukan terlebih dahulu agar dapat diketahui bagaimana bentuk dari perilaku dinamiknya. Kemudian, teknik optimasi desain digunakan untuk meminimalkan perpindahan radial pada spindle yang melibatkan parameter seperti diameter poros, karakteristik dinamik bearing, kecepatan kritis dan amplitudo respon unbalance, selain itu juga untuk mendapatkan diameter poros, kekakuan serta redaman bearing yang seoptimal mungkin. Hasil simulasi numerik telah menunjukkan bahwa dengan mengoptimumkan diameter poros, kekakuan bearing dan redaman bearing, kesalahan radial dari spindle dapat dikurangi. Spindle dengan kesalahan radial sekitar 4µm dapat dikompensasi dengan kepresisian menjadi 2 µm.

***Kata kunci**: kesalahan radial, metode elemen hingga, optimum desain, spindle gerinda presisi tinggi.*

Abstract

In this paper, radial displacement error of a high precision spindle grinding caused by unbalance force was studied. The spindle shaft is considered as a flexible rotor supported by two pairs of angular contact ball bearing. The finite element method (FEM) have been adopted for obtaining the spindle equation motion. In this study, firstly, natural frequencies, critical frequencies and amplitude of unbalance response caused by residual unbalance are determined in order to investigate the spindle behaviors. Further more, an optimization design technique is conducted to minimize radial displacement of the spindle which considers shaft diameters, dynamic characteristics of the bearings, critical frequencies and amplitude of the unbalance response, and computes optimum spindle diameter and stiffness and damping of the bearings. Numerical simulation results show that by optimizing the shaft diameters, and stiffness and damping in the bearings, radial displacement of the spindle can be reduced. A spindle about 4 µm radial displacement error, can be compensated with 2 µm accuracy.

***Keywords**: finite element method, high precision spindle grinding, optimization, radial displacement error.*

INTRODUCTION

Precision spindles are widely used in high precision grinding machine tools. Therefore, a higher machining accuracy is achieved by using these spindles (Rowe, 1967). The advantage of using precision bearing, encouraged machine tool engineers to contribute for development in technology of these spindle bearings. The fundamental methods for designing machine tool spindles can be found in Ref. (Lopez de Lacalle and Lamikiz, 2009). An important function when employing spindles equipped with angular contact bearings, arises from error correction capability. In this article the radial displacement error correction of the spindle was studied due to unbalance mass of the grinding wheel, which is influenced by spindle diameter and the stiffness and damping of angular contact bearings. Many papers have reported that the system parameters such as the diameter of the shaft, stiffness of the shaft and the dynamic coefficients of the bearings, the radial displacement of the shaft could be reduced (Choi and Yang, 2000; Yang et al., 2005;

Straub et al., 2007). For example a spindle with an elliptic manufacturing error of 20 µin, has a radial displacement about 0.2 µin (Levesque, 1965) when supported by an optimized bearing. The high stiffness in the bearings can be achieved by increasing of the initial preload (Alfares and Elsharkawy, 2003), enabling an optimal design to be achieved as in Ref. (Aleyaasin et al., 2000). However, further investigations show that for an optimal performance not only the stiffness parameters of the bearing must be increased, but the bearing damping also should be adjusted (Ozawa, 1994).

In this paper a high precision spindle shaft is modeled as a flexible rotor supported by two sets of angular contact ball bearing. Finite element method (FEM) is employed to built the spindle equation motion in order to describe the spindle dynamic. In this study, natural frequencies, critical speeds and amplitude of unbalance response caused by residual unbalance are determined in order to investigate the spindle behaviors.

An optimization design technique is implemented in order to minimize the radial displacement of the spindle and computes the optimum values of spindle diameter and stiffness and damping of the bearings which considers shaft diameters, dynamic characteristics of the bearings, critical frequencies and amplitude of the unbalance response. Due to the complexity equation of the constraint and objective function, describing the critical speeds and unbalance response, A stochastic search method such as genetic algorithm (GA) (Goldberg, 1989) is employed for the computation of the diameter, stiffness, and damping. The optimum of spindle diameter and stiffness and damping of the bearings are obtained by raising the critical speeds and reducing the unbalance response of the assembly.

The simulation results show that by optimizing the shaft diameters, and stiffness and damping in the bearings for the optimum radial displacement, error correction of spindle displacement can be achieved in certain operating speed. As a simulation example result, an initial design of spindle radial displacement has run-out error about 4 µm can be compensated with 2 µm.

METHODS

Rotating spindle model and theory

Generally, the spindle-bearing system is considered as an assembling of the discrete disks and bearings and the spindle segments with distributed mass. In order to obtain an analysis of the complicated spindle-bearing system, the vibrations are calculated based on the procedure of the finite element discretization in many literatures (Lalanne and Ferraris, 1998; Yamamoto and Ishida, 2001; Friswell et al., 2010), the detail of those equations will not be derived here and only the motion equations are shown below. The system equations that describes the behaviour of entire spindle-bearing system are formulated by taking into account the contributions from all elements in the model. The assembled equation of motion with N_e elements in the global coordinates is of the form (Choi and Yang, 2000)

$$M\ddot{q} - C\dot{q} + Kq = F \tag{1}$$

where $M = (M_t + M_r)$ is the global mass matrix, M_t, M_r are the translational and rotational mass matrices, $C = (-\Omega G + C_b)$, $K = (K_b + K_s)$ are the damping and stiffness matrices, G is a gyroscopic matrix, K_b, C_b are the stiffness and damping matrices of the bearing, and F is a force vector, respectively.

Analysis of eigenvalue

In order to obtain the natural frequency of system, then eigenvalue must be solved and expressed by Eq. (1), the system equation can be set as state variable vector.

$$A\dot{x} + Bx = 0 \tag{2}$$

where the matrices of A, B, and displacement x consist of element matrices given as

$$A = \begin{bmatrix} M_G & C_G \\ 0 & I \end{bmatrix}, \quad B = \begin{bmatrix} 0 & K_G \\ -I & 0 \end{bmatrix}, \quad x = \begin{bmatrix} \dot{q} \\ q \end{bmatrix}$$

For assuming harmonic solution $x = x_0 e^{\lambda t}$ of Eq. (2), the solution of an eigen-value problem is

$$(A\lambda + B)x_0 = 0 \tag{3}$$

where λ is the eigenvalue. The eigenvalues are ussually complex eigenvalues and conjugate roots.

$$\lambda_k = \alpha_k \pm i\omega_k \tag{4}$$

where α_k and ω_k are the stability factor of growth and the *k* mode of damped frequencies, respectively.

Analysis of steady state unbalance response

The forces of unbalance mass (F) which is shown in Eq. (1) can be expressed as:

$$F = F_u \Omega^2 e^{i\Omega t} \quad (5)$$

where F_u is independent of time and rotating speed. The steady-state response due to unbalance mass is considered to be as the form

$$A = A_u e^{i\Omega t} \quad (6)$$

Substituting Eqs. (5) and (6) into (1), the equation can be expressed as

$$(K - \Omega^2 M + i\Omega C)A_u = F_u \Omega^2 \quad (7)$$

By solving Eq. (7) for A_u, the steady-state response can be obtained.

Optimization procedure

The formulation model of optimization for radial displacement reduction problem can be considered as vibration level optimization problem. The optimum values of spindle diameter and the stiffness-damping of bearings are obtained by raising the critical speeds and reducing the unbalance response. For the formulation model, the objective function is to minimize the spindle mass M (Q) and the inequality constraints are subject to the non-linear function of critical speeds and unbalance responses. In this work, spindle diameter, and stiffness and damping of the bearings were selected as the design variables. As we have described above, the formulation model can be expressed as follows:

$$\begin{aligned} &\text{Minimize mass } M(Q) \\ &\Omega_m(Q) \geq \Omega_m^*, \\ &A_m(\Omega_m) \leq A_m^*, \\ &Q_L \leq Q \leq Q_U. \end{aligned} \quad (8)$$

where Ω_m and A_m (m = number of mode) are the new values of critical speeds and unbalance responses for the optimum model, and Ω_m^* and A_m^* are the target constraint values of critical speeds and unbalance response for the initial model. Therefore, it means that the critical speeds, Ω_m, should be increased above given initial values Ω_m^*, and decreasing the unbalance response, A_m, below the given values A_m^*. Moreover, the upper Q_U and lower Q_L bounds on the design variables are set due to manufacturing constraint and to prevent critical stress.

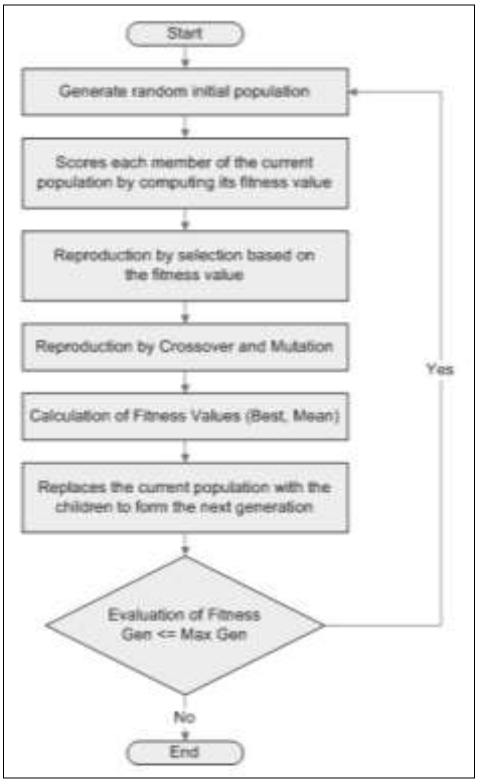

Figure 1. Flowchart of genetic algorithm

Table 1. Searching strategy (GA)

Strategy of input parameter	Description of values
Population size	20
Scaling function	Rank
Selection function	Stochastic uniform
Elite count	2
Crossover fraction	80%
Mutation probability	Constrai dependent
Constraint tolerance	1.10^{-6}
Max number of generation	100

Due to the non-linearity and the complexity functions of critical speeds and unbalance responses, the derivative of these functions are difficult to obtain. Therefore, a stochastic search optimization approach without derivatives such as genetic algorithm (GA) is employed to solve the model of optimization, which performed in MATLAB optimization Toolbox.

Table 1 shows the strategy of input parameter for performing process of genetic algorithm. The flowchart process of genetic algorithm for searching the optimum values of objective function and design variables are described in Fig. 1.

RESULTS AND DISCUSSION

In order to illustrate how the vibration level optimization design technique can be used to minimize the radial displacement of the spindle, a numerical simulation of example was done. A schematic of the spindle finite element model is shown in Fig. 2. In this case, the spindle shaft is modeled into 17 beam elements with a node at both ends of the shaft element. The mass of grinding wheel and pulley can be considered as four elements of the rigid disk which are located at node 1, 14, 15 and 16. In addition, the two sets of bearing are located at node 5 and 12, and the residual unbalance is assumed to occur at node 1.

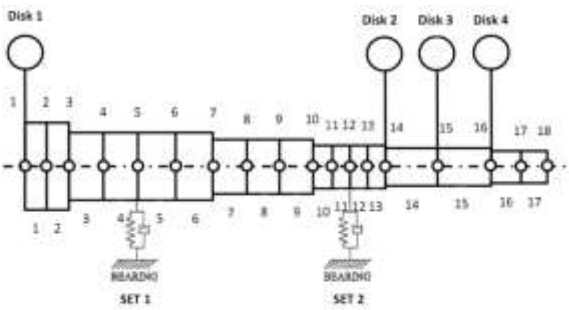

Figure 2. Discretization model of spindle

In the case of vibration level optimization, diameters of shaft element, d_n, (n = element number), and stiffness and damping of the bearings, K_m , C_m , (m = 1,2) are chosen as design variables. Thus, the design variable Q for the spindle models can be written as follows:

$$Q = [d_1, d_2, . , d_{17}, K_1, K_2, C_1, C_2] \qquad (9)$$

Due to the bearing dimension constraint, avoiding the critical stress, and the stability of optimization process is ensured, then the upper and lower values on the diameters of shaft need to be set. The lower and upper bounds on the diameter of the shaft elements are given by Q_L = 0.017 m and Q_U = 0.106 m except in the vicinity of the bearings there is no change of the shaft diameter due to limitations of the bearing size.

For solving the optimization problem, the first is to determine the critical speeds in the main concern of operating speeds range, and then proceed to calculate the magnitude of the unbalance response caused by these critical speeds. These two things will give an overview about the vibration level of system behaviour, and the responses with high amplitude chosen as a target value of the optimization process in which the amplitude needs to be reduced.

Initial simulation results show that, the spindle has two forward modes of the two first critical speeds, which are first forward mode Ω_{1F} = 11910 rpm and second forward mode Ω_{2F} = 21120 rpm, respectively. Due to the first forward mode has a small modal damping ratio (ζ_{1F} = 0.05), it may lead to a very high response peak as illustrated in Fig. 3. The values of critical speeds and maximum amplitude vibration at the first forward mode are

$$\Omega_{1F}^{(0)} = 11910 \text{ rpm}, \quad A_{1F}^{(0)} = 5.032.10^{-5} \text{ m}$$

For the optimization procedure, by substituting the original model values into Eq. (8), re-arranged can be written as

$$\begin{aligned} &\text{Minimize mass } M(Q) \\ &\Omega_m(Q) \geq \Omega_m^* = \Omega_{1F}^{(0)}, \\ &A_m(\Omega_m) \leq A_m^* = A_{1F}^{(0)}, \\ &0.017 \leq Q \leq 0.106. \end{aligned} \qquad (10)$$

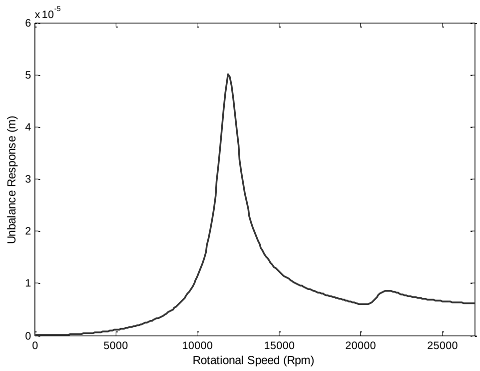

Figure 3. Unbalance response of the spindle

The numerical values are initial mass m = 3.6 kg, operating speed Ω = 8000 rpm and initial values are d_1~d_2 = 88 mm, d_3~d_6 = 70 mm, d_7~d_9 = 64.5 mm, d_{10}~d_{13} = 60 mm, d_{14}~d_{15} = 54.5, d_{16}~d_{17} = 50.4, $K_1 = 1.911\times10^8$ N/m, $K_2 = 2.476\times10^8$ N/m, $C_1 = 191.1\times10^2$ N.s/m and $C_2 = 247.6\times10^2$ N.s/m. The initial radial displacement in Eq. (7) is A_m = 4.252 µm when the allowance residual unbalance (1 gr.mm) is applied to the grinding wheel. In Figs. 4 and 5 the time response of the displcement in the y and z axis direction and absolute displacement of the spindle respectively before optimization are shown.

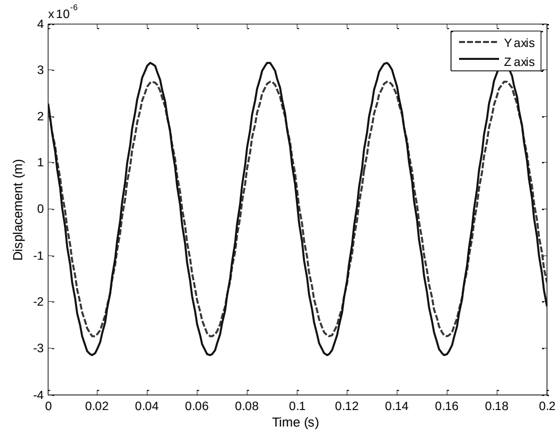

Figure 4. Radial displacement in the y and z axis

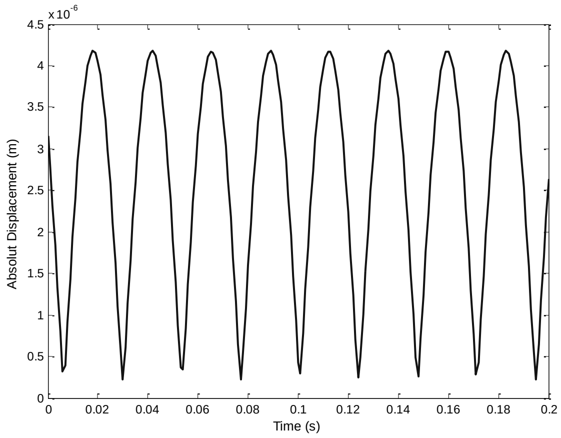

Figure 5. Absolute radial displacement of spindle (before optimization)

The optimum values of the spindle diameter and the stiffness and damping of the bearings which minimize the radial displacement of the spindle are shown in Tables 2 and 3.

Table 2. Shaft diameter of the spindle

Diameter	Initial values	Optimum values
d_1~d_2	88	88
d_3~d_6	70	70
d_7~d_9	64.5	60
d_{10}~d_{13}	60	60
d_{14}~d_{15}	54.5	54.5
d_{16}~d_{17}	50.4	17

Table 3. Characteristics dynamic of the bearing

Bearing	Initial values	Optimum values
Stiffness (N/m)		
K_1	1.911×10^8	3.797×10^8
K_2	2.476×10^8	3.240×10^8
Damping (N.s/m)		
C_1	191.1×10^2	218.2×10^2
C_2	247.6×10^2	267.7×10^2

The comparison of unbalance response at the grinding wheel due to the residual unbalance before and after optimization is shown in Fig. 6. It can be seen that the spindle diameter and the stiffness and damping of the bearings are effective to increase the critical speed and to decrease the amplitude of the unbalance response at first mode. The total shaft mass, 1st critical speed and unbalance response for the initial and optimum model which is optimized by genetic algorithm (GA) are presented in Table 4.

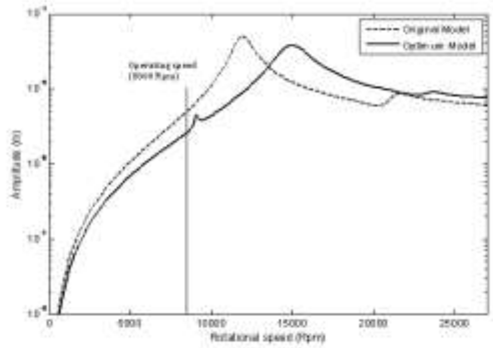

Figure 6. Comparison of unbalance respon

Table 4. Opimum values computed by GA

	Initial values	Optimum values
Total mass of shaft (kg)	3.6	3.36
1st critical speed (rpm)	11910	14838
Unbalance response (m)	5.032×10^{-5}	3.691×10^{-5}

The simulation result shows that, after optimizing the spindle shaft, and adjusting the bearings to an optimal stiffness and damping, which the allowance residual unbalance (1 gr.mm) is applied to the grinding wheel, therefore the maximum radial displacement of the spindle for operating speed at 8000 rpm would be A_m = 2.328 µm as illustrated in Fig. 6. In Figs. 7 and 8 the time response of the displcement in the y and z axis direction and absolute displacement of the spindle after optimization are shown respectively. In Fig. 8 the absolute displacement of the spindle shows a great decrease, about 45.2% in the amplitude when compared with Fig. 5. This certainly can improve the accuracy of the machining process.

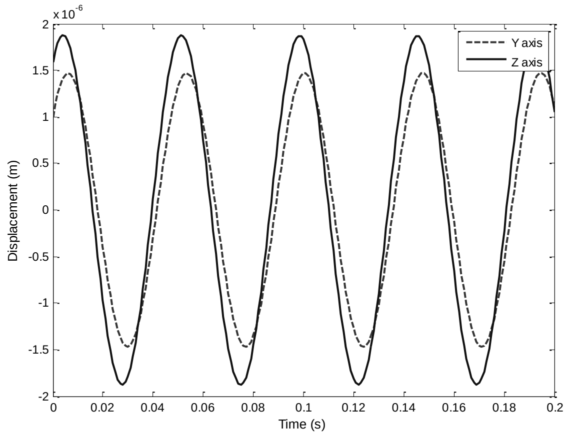

Figure 7. Radial displacement in the y and z axis (after optimization)

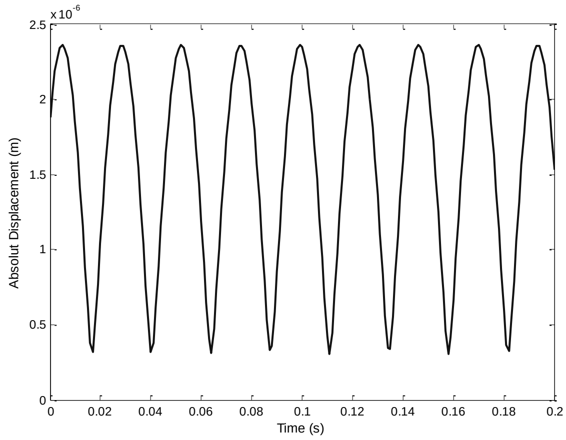

Figure 8. Absolute radial displacement of spindle-bearing (after optimization)

CONCLUSION

An optimization design technique such as vibration level optimization has been implemented successfully in order to minimize the radial displacement of the spindle. In this study, vibration level optimization model was applied to find the spindle diameter and the stiffness-damping of the bearings optimum values by raising the critical speeds and reducing the unbalance response. The objective of this optimization problem is only to minimize the spindle mass under critical speed and unbalance response constraint. Simulation results show that the radial displacement of the spindle at operating speed 8000 rpm was reduced satisfactory, about 45.2% when optimizing the spindle shaft, and adjusting the dynamic characteristics of the bearing to an optimal stiffness and damping. This certainly can improve the accuracy of the machining process.

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