ALTERNATING DIRECTION IMPLICIT METHOD FOR SOLVING MODIFIED REYNOLDS EQUATION IN A LUBRICATED SLIDING CONTACT

M. Tauviqirrahman[1), R. Ismail[1), Jamari[2), and D. J. Schipper[1)

1) Laboratory for Surface Technology and Tribology, Faculty of Engineering Technology, University of Twente, Drienerloolaan 5, Postbus 217, 7500 AE, Enschede, The Netherlands

E-mail : mtauviq99@yahoo.com

2) Department of Mechanical Engineering, University of Diponegoro

Jl. Prof. Sudharto, UNDIP Tembalang, Semarang, Indonesia

Abstract

Reynolds lubrication theory assumes that there is no-slip boundary on the interfaces between solid and lubricant. However, during the past few years, slip boundary has been observed in micro/nanometer gap. In this paper, numerical studies of the Reynolds model with slip boundary are presented for investigating the effect of slip boundary on the pressure distribution in sliding contact. The modified Reynolds equation is solved numerically by finite difference method with alternating direction implicit algorithm. Line by line algorithm was applied to overcome multidimensional direction. The simulation results indicate that the two-dimensional modified Reynolds equation can predict the dynamic of lubrication process with boundary slip in sliding contact.

Keywords : *Slip boundary, finite difference method, hydrophobic, hydrophilic*

Introduction

For hundreds of years, the classical lubrication mechanics in both scientific research and engineering application has been based on the frame of the Reynolds equation with no-slip assumption, i.e. the fluid velocity matches the velocity of the boundary solid. While the generally-accepted no-slip boundary condition has been validated experimentally for a number of macroscopic flow, it remains that the assumption is not based on the physical principles. The success of applying the no-slip boundary condition to many engineering areas cannot reflect the accuracy of the boundary condition but may reflect the insensitivity of an experiment to the partial-slip boundary condition (Craig *et al*., 2001).

A small amount of boundary slip might not dramatically affect the macroscopic measurement of fluid flow. However, a partial or large slip might have large influence on the micro/nano-scale flows. Recently, several researchers have suggested that the no-slip boundary condition may not be suitable at both the micro- and nano- scale (Craig *et al*., 2001, Pit *et al*., 2000, Zhu and Granick, 2001, Tretheway and Meinhart, 2002). It indicates the proper condition depends both on the characteristic length scale of the flow and the chemical and physical properties of the solid surface. During recent years, with the advancement of the experimental techniques, such as nano-particle image velocimetry (NPIV), atomic force microscope (AFM) and surface force apparatus (SFA), boundary slip of a liquid occurs not only at hydrophobic surface (Craig *et al*., 2001, Pit *et al*., 2000, Zhu and Granick, 2001), but also at a hydrophilic surface (Bonaccurso *et al*., 2003). Boundary slip occurs not only in a polymer flow (Wu and Ma, 2005), but also in a hydrodynamic (Kaneta *et al*., 1990) and elastohydrodynamic (Cieplak *et al*., 2001) lubrication. Therefore, the slip evidence has been generally accepted and for certain cases the no-slip boundary condition is not valid.

In relation to the slip boundary, for nearly two hundred years ago, Navier's hypothesis (Navier, 1823) proposed that the velocity is proportional to the shear rate at the wall with the slip length, β , a constant.

$$u_s = \beta\eta \frac{\partial u}{\partial y} \tag{1}$$

If β = 0 the generally assumed no-slip boundary condition is obtained. If β = finite, fluid slip occurs at the wall, but its effect depends upon the length scale of the flow. The boundary condition is evaluated at the surface. The slip length is the distance behind the interface at which the liquid velocity extrapolates to zero. For a pure shear flow, the slip length β can be interpreted as the fictitious distance below the surface where the no-slip boundary condition would be satisfied (Fig. 1).

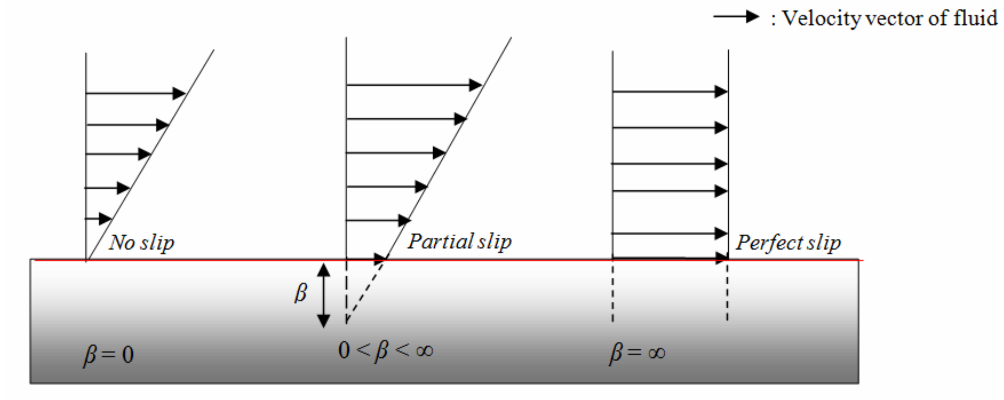

Fig. 1. Interpretation of the slip length, β (Lauga *et al*., 2007).

Application of slip may not be restricted to the small in scale, but can be applied to the wider area. Salant and Fortier (2005) conducted a numerical analysis of a finite slider bearing with a heterogeneous slip/no-slip surface by means a modified slip length model and found that such a bearing can provide a high load support but low friction drag. Wu and Sun (2006) presented a general numerical method for the numerical solution of wall slip and free boundary condition using a finite element method together with a quadratic programming technique. It was found that the slip boundary dramatically affects generation of the hydrodynamic pressure. Large slip even causes a null hydrodynamic pressure in a sliding contact. Generally speaking, the slip boundary plays an important role in hydrodynamic pressure generation of the fluid in a solid gap with relative motion at both the macro- and micro-scale.

In the present study, the investigation of the slip boundary phenomenon is examined by means of a numerical analysis using alternating direction method. The focus of the analysis is the effect of the slip boundary on the pressure distribution in hydrodynamic lubrication. In order to simulate the pressure distribution, a computer code has been developed using the set of equations in the following section.

Mathematical model

The derivation of classical Reynolds equation is based on the assumption of no-slip between the lubricant and the contacting surfaces, i.e., the lubricant velocities at the surfaces are set equal to the surface velocities. In this paper, the governing equation for a liquid film in the two-dimensional flow, taking account of the slip boundary, could be derived. The following assumptions in order to simplify the mathematical modeling and computation in micro-gap are used in the present analysis. They are (1) the fluid is Newtonian, (2) the fluid flow in the film region is laminar, (3) inertial forces are negligible compared to viscous terms, (4) the body forces are assumed to be absent, (5) viscosity is constant, and (5) the pressure is constant across the film thickness. The result is a modified version of the Reynolds equation. This equation can be written as follow:

$$\frac{\partial}{\partial x}\left\{\frac{h^3}{12\eta}\frac{\partial p}{\partial x}\left(1+\frac{3\beta\eta}{h+\beta\eta}\right)\right\}+\frac{\partial}{\partial y}\left\{\frac{h^3}{12\eta}\frac{\partial p}{\partial y}\left(1+\frac{3\beta\eta}{h+\beta\eta}\right)\right\}=\frac{\partial}{\partial x}\left\{\frac{u_w h}{2}\left(1+\frac{\beta\eta}{h+\beta\eta}\right)\right\} \quad (2)$$

or, in dimensionless form

$$\frac{\partial}{\partial X}\left\{H^3\frac{\partial \hat{P}}{\partial X}\left(1+\frac{3B}{H+B}\right)\right\}+L^2\frac{\partial}{\partial Y}\left\{H^3\frac{\partial \hat{P}}{\partial Y}\left(1+\frac{3B}{H+B}\right)\right\}=U\frac{\partial}{\partial X}\left\{H\left(1+\frac{B}{H+B}\right)\right\} \quad (3)$$

where

$$H=\frac{h}{h_0},\ X=\frac{x}{L_x},\ Y=\frac{y}{L_y},\ \hat{P}=\frac{p}{p_a},\ B=\frac{\beta\eta}{h_0},\ L=\frac{L_x}{L_y},\ U=\frac{6u_w\eta L_x}{p_a h_0^2}$$

Since full film condition are assumed, the entire load per width unit is carried by lubricant film and the calculation is simply an integration of the lubricant film pressure. The load carrying capacity is defined as the integral of the pressure profile over the surface area, and in terms of dimensionless quantities:

$$\hat{W}=\int_0^1\int_0^1 \hat{P}(X,Y)\,dX\,dY \tag{4}$$

Numerical Procedure

The physical configuration of the problem under consideration is shown in the Fig. 2. The lower surface without slip moves with a sliding velocity u_w. Surface 2 is the stationary surface. The slip boundary is applied everywhere on surface 2.

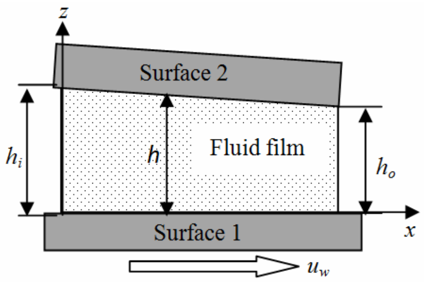

Fig. 2. Schematic of a sliding micro-gap with slip boundary

In this study, the height of the fluid film separating the two surfaces, is assumed to be a linear function of *x*. The fluid through the converging wedge can result in hydrodynamic pressure generation. The top surface, labeled as Surface 2 in Fig. 4 is stationary while the bottom surface, Surface 1, is moving at a sliding speed, u_w. In the present model the pressure is assumed as atmospheric pressure on all external edges.

The fluid velocity in the upper surface is proportional to the surface shear stress. The relevant boundary conditions for the velocity components are:

(1) at the lower surface (z = 0) $\quad u_x = u_w,\quad u_y = 0$

(2) at the upper surface (z = *h*) $\quad u_x=-\beta\eta\dfrac{\partial u_x}{\partial z},\quad u_y=-\beta\eta\dfrac{\partial u_y}{\partial z}$

where β is the slip length. If the slip length is set equal to zero the above boundary condition reduces to the traditional no-slip case.

By employing the discretization scheme, the computed domain are divided into a number control volumes using a grid with uniform mesh size, ΔX and ΔY. The calculational domain is $0 \le X \le L_x$ and $0 \le Y \le L_y$. The mesh size is $\Delta X = L_x/n$ and $\Delta Y = L_y/n$, and n+1 is the number of grid points. The uniform grid is applied on slip face surface. The 50 x 50 meshes are employed in the computational domain. The grid independency was validated by various numbers of size meshes. Whether the meshes number was above 50 x 50, the simulation results were almost the same. But obviously the computational cost increased. Considering the processing time limitation, 50 x 50 meshes was adopted for all simulation cases.

Solution of Algebraic Equation

The dimensionless Reynolds equation, Eq. (3) is solved numerically using finite difference equations. The modified Reynolds equation is discretized using a first order central difference and solved using the alternating direction implicit method (ADI). In this method one line of nodes is examined at time, while nodes on neighboring lines are assumed to be known values and remain constant (Fig. 3). The problem along a single line can be solved with a Tri-Diagonal Matrix Algorithm (TDMA). The solution proceeds along all lines in one direction then switches to lines in the perpendicular direction (Versteg and Malalasekera, 1995).

- ● Points at which values are calculated.
- ■ Points at which values are considered to be temporarily known
- **x** Known boundary values

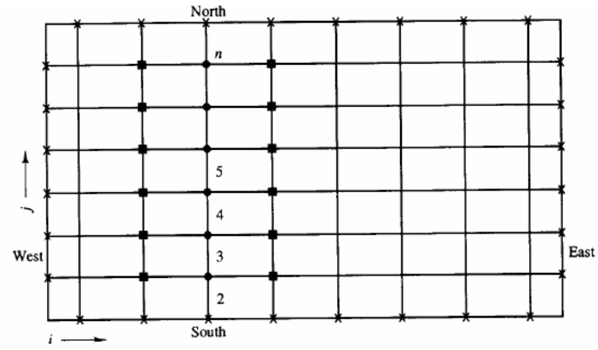

Fig. 3. Sweeping representation of the line by line method

Results and Discussions

Computations have been made for a sliding micro-gap with slip or no-slip surface. The below figure contains pressure distribution for a micro-gap with a dimensionless length to width ratio $L_x/L_y = 1$, dimensionless slip length B = 100, and dimensionless sliding velocity U = 100. Fig 4(a) shows the pressure distribution for the no-slip surface, while Fig. 4(b) shows the pressure distribution for the surface 2 with slip applied. The load carrying capacity, $\hat{W}$ computed for the no-slip boundary is 0.6258. For a surface of the same configuration but with any slip (B = 100) the corresponding load support is 0.3120. In this instance, the load support gained by the addition of slip decrease about in half compared to that of the no-slip surface. This result is in a good agreement with the works of Wu & Sun (20). They found that the slip boundary produces a low hydrodynamic pressure in lubrication film. It also indicates that the hydrodynamic of a lubrication film between two poorly wetting (hydrophobic) surfaces is entirely different from between two no-slip surfaces. The slip boundary shows a disadvantage over the surface if applied everywhere on the surface.

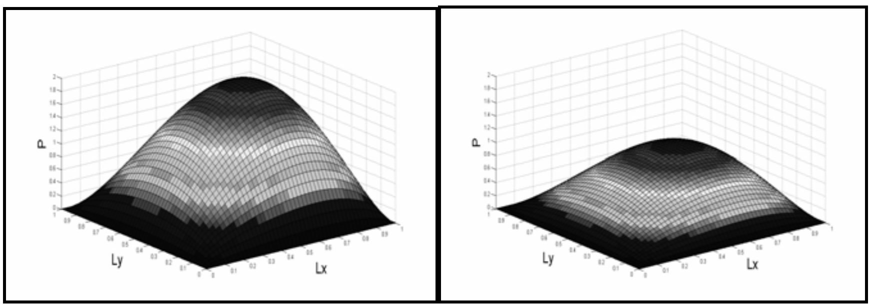

Fig. 4. Pressure distribution for (a) no-slip analysis, (b) slip boundary analysis

In this study, the effect of changing driving velocity is also evaluated with respect to the load carrying capacity, W (Eq. 4). Figure 5 shows the effect of changing the dimensionless sliding velocity, U, on the load carrying capacity. Load is found to have a linear variation with velocity. When sliding velocity is absent (U = 0), there is zero load support for two configurations. The load carrying capacity supported by the surface with slip boundary shows a lower load than those without slip. It achieves a load carrying capacity that is 0.5 times that of the surface with the no-slip boundary.

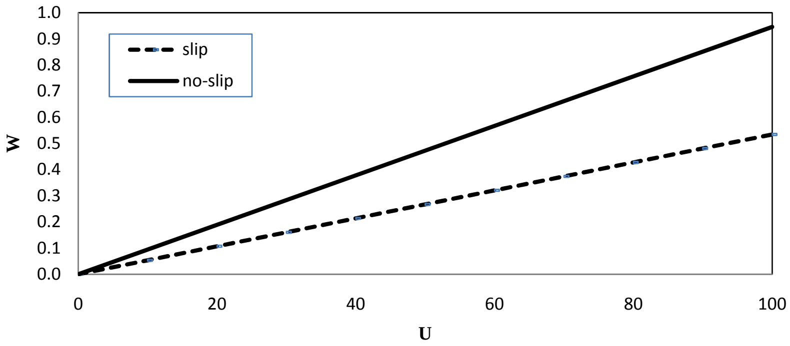

Fig. 5. Effect of the sliding velocity on the load carrying capacity

Conclusions

Set of partial differential equations of modified Reynolds equation can be solved numerically by using finite difference method with alternating direction implicit methods (ADI) algorithm. Line by line method for sweeping of multidimensional direction can successfully give solution of algebraic equations. Numerical studies reported in this paper show that the hydrodynamic of a lubrication film confined between a no-slip surface and a slip surface differs entirely from that of the film confined between two no-slip surfaces. It is found that a slip boundary produces lower hydrodynamic pressure in a lubricant film between two solid surfaces. The hydrodynamic of the lubrication film between slip surfaces may bring us a new idea in both the scientific research and engineering design for MEMS (micro-electro-mechanical system) and micro-fluidic based devices. There is a need to make an optimized heterogeneous pattern for wider MEMS application.

Nomenclature

β = slip length

η = viscosity

h = film thickness

L = aspect ratio, L_x/L_y

L_x = length of surface in x-direction

L_y = length of surface in y-direction

u_w = sliding velocity of moving surface

u_x = X component of velocity

u_y = Y component of velocity

p = pressure

p_a = atmospheric pressure

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